

# Strong Converse for a Degraded Wiretap Channel via Active Hypothesis Testing

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**Abstract**—We establish an upper bound on the rate of codes for a wiretap channel with public feedback for a fixed probability of error and secrecy parameter. As a corollary, we obtain a strong converse for the capacity of a degraded wiretap channel with public feedback. Our converse proof is based on a reduction of active hypothesis testing for discriminating between two channels to coding for wiretap channel with feedback.

## I. INTRODUCTION

We consider secure message transmission over a wiretap channel  $W : \mathcal{X} \rightarrow \mathcal{Y} \times \mathcal{Z}$  with noiseless, public feedback. For each transmission  $x \in \mathcal{X}$  over  $W$ , the receiver observes a random output  $Y \in \mathcal{Y}$  and an eavesdropper observes a correlated side-information  $Z \in \mathcal{Z}$ , with probability  $W(Y, Z|x)$ . Furthermore, the receiver can send a feedback to the transmitter over a noiseless channel. However, the feedback channel is public and any communication sent over it is available to the eavesdropper. The transmitter seeks to send a message  $M$  to the receiver without revealing it to the eavesdropper. For a given probability of error  $\epsilon$  and a given secrecy parameter  $\delta$ , what is the maximum possible rate  $C_{\epsilon, \delta}$  of a transmitted message?

For a degraded wiretap channel  $W$  with no feedback, the wiretap capacity  $C = \inf_{\epsilon, \delta} C_{\epsilon, \delta}$  was established in the seminal work of Wyner [19] where it was shown that

$$C = \max_{P_X} I(X \wedge Y | Z).$$

The capacity of a general wiretap channel was established in [3]. Extensions to wiretap channels with general statistics were considered in [4]. The model with feedback considered here was introduced in [8] where it was noted that the availability of a noiseless feedback can enable positive rates of transmission over a wiretap channel with zero capacity (see, also, [10]). However, the wiretap capacity with feedback remains unknown in general;  $\max_{P_X} I(X \wedge Y | Z)$  constitutes an upper bound on it.

In this paper, we establish a *strong version* of this bound and show that for  $\epsilon + \delta < 1$

$$C_{\epsilon, \delta} \leq \max_{P_X} I(X \wedge Y | Z),$$

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thereby characterizing  $C_{\epsilon, \delta}$  for all  $0 < \epsilon, \delta < 1$  for a degraded wiretap channel. A partial strong converse for a degraded wiretap channel was established in [11] for a restricted range of  $\epsilon, \delta$ . Another strong converse for a degraded wiretap channel for the case when  $\delta \rightarrow 0$  was established, concurrently to this work, in [15]. In this work, we show a strong converse for all values of  $\epsilon$  and  $\delta$ .

Our proof relies on a slight modification of a recent reduction of hypothesis testing to secret key agreement shown in [17], [18]. Specifically, we show that a wiretap channel code yields an active hypothesis test for distinguishing between two channels [6]. Consequently, the rate of a wiretap code is bounded above by the rate of the optimum exponent of the probability of error of type II for discriminating a channel  $W$  from another channel  $V$  such that  $V(y, z|x) = V_2(z|x)V_1(y|z)$ , given that the probability of error of type I is less than  $\epsilon + \delta$ . This gives an upper bound on the length of a wiretap code, which leads to the strong converse upon using the characterization of the optimal exponent for channel discrimination derived in [6]. This approach is along the lines of *meta-converse* of [13], where a reduction of hypothesis testing to channel coding was used to establish a finite-blocklength converse for the channel coding problem (see, also, [12] and [5, Section 4.6]).

Our main result is given in the next section. Section III and IV contains a review of relevant results in binary hypothesis testing and secret key agreement, respectively. The final section contains a proof of our main result.

## II. MAIN RESULT

We describe a generalization of the classic wiretap channel coding problem [19], [3] that was considered in [8], [10], [1], where, in addition to transmitting over the wiretap channel, the terminals can communicate using a noiseless, public feedback channel from the receiver to the transmitter.

A wiretap code for a discrete<sup>1</sup> memoryless wiretap channel  $W : \mathcal{X} \rightarrow \mathcal{Y} \times \mathcal{Z}$  with feedback consists of (possibly randomized) encoder mappings  $e_t : \{1, \dots, N\} \times \mathcal{F}^t \rightarrow \mathcal{X}$ ,  $1 \leq t \leq n$ , feedback mappings  $f_t : \mathcal{Y}^t \rightarrow \mathcal{F}$ ,  $0 \leq t \leq n-1$ , and a decoder  $d : \mathcal{Y}^n \rightarrow \{1, \dots, N\}$ . For a random message  $M \sim \text{unif}\{1, \dots, N\}$ , the protocol begins with a feedback  $F_0$  from the receiver at  $t = 0$ . Subsequently, at each time instance  $1 \leq t \leq n-1$  the transmitter sends  $X_t = e_t(M, F^{t-1})$  and the channel outputs  $(Y_t, Z_t)$

<sup>1</sup>The restriction to discrete alphabet is cosmetic. Our results apply to channels with continuous alphabet. In particular, our strong converse holds for the Gaussian wiretap channel [9].

with probability  $W(Y_t, Z_t | X_t)$ . The receiver observes  $Y_t$  and sends feedback  $F_t = f_t(Y^t)$ , and the eavesdropper observes  $Z_t$ . The protocol stops with a final transmission  $X_n = e_n(M, F^{n-1})$  over the channel and the subsequent decoding  $\hat{M} = d(Y^n)$  by the receiver. We denote by  $\mathbf{F}$  the overall feedback communication  $F_0, \dots, F_{n-1}$ .

The mappings  $(\{e_t\}_{t=1}^n, \{f_t\}_{t=0}^{n-1}, d)$  constitute an  $(N, n, \epsilon, \delta)$  wiretap code if

$$P(M \neq \hat{M}) \leq \epsilon,$$

and

$$\|P_{MZ^n\mathbf{F}} - P_M \times P_{Z^n\mathbf{F}}\|_1 \leq \delta,$$

where  $\|P - Q\|_1$  denotes the variation distance between  $P$  and  $Q$  given by

$$\|P - Q\|_1 = \frac{1}{2} \sum_x |P(x) - Q(x)|.$$

A rate  $R > 0$  is  $(\epsilon, \delta)$ -achievable if there exists an  $(\lfloor 2^{nR} \rfloor, n, \epsilon, \delta)$  wiretap code for all  $n$  sufficiently large. The  $(\epsilon, \delta)$ -wiretap capacity  $C_{\epsilon, \delta}$  is the supremum of all  $(\epsilon, \delta)$ -achievable rates.

Our main result is an upper bound on  $C_{\epsilon, \delta}$

**Theorem 1.** *For  $0 \leq \epsilon, \delta$  with  $\epsilon + \delta < 1$ , the  $(\epsilon, \delta)$ -wiretap capacity is bounded above as*

$$C_{\epsilon, \delta} \leq \max_{P_X} I(X \wedge Y | Z).$$

For the special case of a degraded wiretap channel  $W$  with  $W(y, z|x) = W_1(y|x)W_2(z|y)$ , Theorem 1 yields a strong converse for wiretap capacity.

**Corollary 2.** *For a degraded wiretap channel  $W$ ,*

$$C_{\epsilon, \delta} = \begin{cases} \max_{P_X} I(X \wedge Y | Z), & 0 < \epsilon < 1 - \delta, \\ \max_{P_X} I(X \wedge Y), & 1 - \delta \leq \epsilon < 1. \end{cases}$$

*Proof.* For  $0 < \epsilon < 1 - \delta$ , the result is an immediate corollary of Theorem 1 and [19]<sup>2</sup>. For  $1 - \delta \leq \epsilon < 1$ , the converse follows from the strong converse for the capacity of a DMC with feedback (cf. [14]). Moving to the proof of achievability, it suffices to restrict to  $\epsilon + \delta = 1$ . For this case, achievability follows by randomizing between an  $(\epsilon_n, 1)$  wiretap code,  $\epsilon_n \rightarrow 0$  as  $n \rightarrow \infty$ , and a  $(1, 0)$  wiretap code – the randomizing bit is communicated as the public feedback  $F_0$  by the receiver<sup>3</sup>.  $\square$

As a preparation for the proof of Theorem 1 given in Section V, we review some results in hypothesis testing and secret key agreement in the next two sections.

### III. HYPOTHESIS TESTING

Consider a simple binary hypothesis testing problem with null hypothesis  $P$  and alternative hypothesis  $Q$ , where  $P$  and

$Q$  are distributions on the same alphabet  $\mathcal{X}$ . Upon observing a value  $x \in \mathcal{X}$ , the observer needs to decide if the value was generated by the distribution  $P$  or the distribution  $Q$ . To this end, the observer applies a stochastic test  $T$ , which is a conditional distribution on  $\{0, 1\}$  given an observation  $x \in \mathcal{X}$ . When  $x \in \mathcal{X}$  is observed, the test  $T$  chooses the null hypothesis with probability  $T(0|x)$  and the alternative hypothesis with probability  $T(1|x) = 1 - T(0|x)$ . For  $0 \leq \epsilon < 1$ , denote by  $\beta_\epsilon(P, Q)$  the infimum of the probability of error of type II given that the probability of error of type I is less than  $\epsilon$ , i.e.,

$$\beta_\epsilon(P, Q) := \inf_{T: P[T] \geq 1 - \epsilon} Q[T],$$

where

$$\begin{aligned} P[T] &= \sum_x P(x) T(0|x), \\ Q[T] &= \sum_x Q(x) T(0|x). \end{aligned}$$

The following result credited to Stein characterizes the optimum exponent of  $\beta_\epsilon(P^n, Q^n)$  where  $P^n = P \times \dots \times P$  and  $Q^n = Q \times \dots \times Q$ .

**Lemma 3.** (cf. [7, Theorem 3.3]) *For every  $0 < \epsilon < 1$ , we have*

$$\lim_{n \rightarrow \infty} -\frac{1}{n} \log \beta_\epsilon(P^n, Q^n) = D(P \| Q),$$

where  $D(P \| Q)$  is the Kullback-Leibler divergence given by

$$D(P \| Q) = \sum_{x \in \mathcal{X}} P(x) \log \frac{P(x)}{Q(x)},$$

with the convention  $0 \log(0/0) = 0$ .

Next, we review a problem of active hypothesis testing where the distribution at each instance is determined by a prior action. Specifically, given two DMCs  $W : \mathcal{X} \rightarrow \mathcal{Y}$  and  $V : \mathcal{X} \rightarrow \mathcal{Y}$ , we seek to design a transmission-feedback scheme such that by observing the channel inputs, channel outputs, and feedback we can determine if the underlying channel is  $W$  or  $V$ . Formally, an  $n$ -length active hypothesis test consist of (possibly randomized) encoder mappings  $e_t : \mathcal{F}^t \rightarrow \mathcal{X}$ ,  $1 \leq t \leq n$ , feedback mappings  $f_t : \mathcal{Y}^t \rightarrow \mathcal{F}$ ,  $0 \leq t \leq n - 1$ , and a conditional distribution  $T$  on  $\{0, 1\}$  given  $X^n, Y^n, \mathbf{F}$ . On observing  $X^n, Y^n, \mathbf{F}$ , we detect the null hypothesis  $W$  with probability  $T(0|X^n, Y^n, \mathbf{F})$  and alternative hypothesis  $V$  with probability  $T(1|X^n, Y^n, \mathbf{F})$ . Analogous to  $\beta_\epsilon(P, Q)$ , the quantity  $\beta_\epsilon(W, V, n)$ , for  $0 \leq \epsilon < 1$ , is the infimum of the probability of error of type II over all  $n$  length active hypothesis tests for null hypothesis  $W$  and alternative hypothesis  $V$  such that the probability of error of type I is no more than  $\epsilon$ .

The following analogue of Stein's lemma for active hypothesis testing was established in [6] (see, also, [14]).

<sup>2</sup>While the secrecy criterion in [19] is different from variational secrecy required here, the achievability result for the latter follows from the results in [2], [4].

<sup>3</sup>Alternatively, the sender can transmit the randomizing bit over the wiretap channel with negligible rate loss.

**Theorem 4** ([6]). For  $0 < \epsilon < 1$ ,

$$\begin{aligned} \lim_{n \rightarrow \infty} -\frac{1}{n} \log \beta_\epsilon(W, V, n) &= \max_{P_X} D(W \| V | P_X) \\ &= \max_x D(W_x \| V_x), \end{aligned}$$

where  $W_x$  and  $V_x$ , respectively, denote the  $x$ th row of  $W$  and  $V$ .

Remarkably, the exponent above is achieved without any feedback, *i.e.*, while feedback is available, it does not help to improve the asymptotic exponent of  $\beta_\epsilon(W, V, n)$ .

#### IV. SECRET KEY AGREEMENT

In this section, we review two party secret key (SK) agreement where parties observing random variables  $X$  and  $Y$  communicate interactively over a public channel to agree on a SK that is concealed from an eavesdropper with access to the communication and a side-information  $Z$ .

Formally, the parties communicate using an interactive communication  $\mathbf{F} = F_1, \dots, F_r$  where  $F_1 = F_1(X)$ ,  $F_2 = F_2(Y, F_1)$ ,  $F_3 = F_3(X, F^2)$ ,  $F_4 = F_4(Y, F^3)$  and so on. A random variable  $K = K(X, \mathbf{F})$  constitutes an  $(\epsilon, \delta)$ -SK if there exists  $\hat{K} = \hat{K}(Y, \mathbf{F})$  such that

$$P(K \neq \hat{K}) \leq \epsilon,$$

and

$$\|P_{KZ\mathbf{F}} - P_{\text{unif}} \times P_{Z\mathbf{F}}\|_1 \leq \delta.$$

The following upper bound on the number of values  $k$  taken by an  $(\epsilon, \delta)$ -SK  $K$  was shown in [17], [18]:

$$\log k \leq -\log \beta_{\epsilon+\delta+\eta}(P_{XYZ}, Q_{XYZ}) + 2 \log \frac{1}{\eta},$$

for all  $0 < \eta < 1 - \epsilon - \delta$ , and all  $Q_{XYZ} = Q_{X|Z}Q_{Y|Z}Q_Z$ . Underlying the proof of this bound is an intermediate reduction argument in [17, Lemma 1] that relates SK agreement to hypothesis testing. We recall this result below.

**Theorem 5** ([17], [18]). For  $0 \leq \epsilon, \delta, \epsilon + \delta < 1$ , let random variables  $K, \hat{K}$ , and  $Z$  be such that  $P(K \neq \hat{K}) \leq \epsilon$  and

$$\|P_{KZ} - P_{\text{unif}} \times P_Z\|_1 \leq \delta,$$

where  $P_{\text{unif}}$  denotes a uniform distribution on  $k$  values. Then, for every  $0 < \eta < 1 - \epsilon - \delta$  and every  $Q_{K\hat{K}Z} = Q_{K|Z}Q_{\hat{K}|Z}Q_Z$ ,

$$\log k \leq -\log \beta_{\epsilon+\delta+\eta}(P_{K\hat{K}Z}, Q_{K\hat{K}Z}) + 2 \log \frac{1}{\eta}.$$

#### V. PROOF OF MAIN RESULT

We present a converse result that applies for every fixed  $n$  and is asymptotically tight, giving the strong converse result of Theorem 1.

**Theorem 6.** For  $0 \leq \epsilon, \delta, \epsilon + \delta < 1$ , given an  $(N, n, \epsilon, \delta)$ -wiretap code, we have

$$\log N \leq -\log \beta_{\epsilon+\delta+\eta}(W, V, n) + 2 \log \frac{1}{\eta},$$

for all  $0 < \eta < 1 - \epsilon - \delta$  and all channels  $V : \mathcal{X} \rightarrow \mathcal{Y} \times \mathcal{Z}$  such that  $V(y, z|x) = V_2(z|x)V_1(y|z)$ .

*Proof of Theorem 1.* Theorem 1 follows from Theorems 6 and 4 upon noting that for  $W(y, z|x) = W_2(z|x)W_1(y|z, x)$

$$\begin{aligned} \min_V \max_{P_X} D(W \| V | P_X) &= \min_{V_1} \max_{P_X} D(W_1 \| V_1 | P_X W_2) \\ &= \max_{P_X} \min_{V_1} D(W_1 \| V_1 | P_X W_2) \\ &= \max_{P_X} D(P_{Y|ZX} \| P_{Y|Z} | P_{ZX}) \\ &= \max_{P_X} I(X \wedge Y | Z), \end{aligned}$$

where  $P_{XYZ}$  is given by  $P_X W$ .  $\square$

We need the following result to prove Theorem 6.

**Lemma 7.** For a wiretap channel  $V : \mathcal{X} \rightarrow \mathcal{Y} \times \mathcal{Z}$  such that  $V(y, z|x) = V_2(z|x)V_1(y|z)$ , a random message  $M$ , and a wiretap code, let  $\hat{M} = d(Y^n)$  and  $\mathbf{F}$  be the corresponding feedback. Then, the induced distribution  $Q_{M\hat{M}Z^n\mathbf{F}}$  satisfies factorization condition

$$Q_{M\hat{M}|Z^n\mathbf{F}} = Q_{M|Z^n\mathbf{F}} \times Q_{\hat{M}|Z^n\mathbf{F}}.$$

*Proof of Lemma 7.* Denote by  $U_x$  and  $U_y$ , respectively, the local randomness at the transmitter and the receiver, and by  $F^t$  the feedback  $(F_0, \dots, F^t)$ . Thus, the encoder mapping  $e_t$  is a (deterministic) function of  $(M, U_x, F^{t-1})$  and the feedback mapping  $f_t$  is a (deterministic) function of  $(Y^t, U_y)$ . The proof entails a repeated application of the fact that conditionally independent random variables remain so when conditioned additionally on an interactive communication (cf. [16]) and is completed by induction. Specifically, note first that  $Q_{MU_x U_y | F_0} = Q_{MU_x | F_0} Q_{U_y | F_0}$  since  $(M, U_x)$  and  $U_y$  are independent and  $F_0$  is an interactive communication. Under the induction hypothesis

$$\begin{aligned} Q_{MU_x X^{t-1} U_y Y^{t-1} | Z^{t-1} F^{t-1}} \\ = Q_{MU_x X^{t-1} | Z^{t-1} F^{t-1}} Q_{U_y Y^{t-1} | Z^{t-1} F^{t-1}}, \end{aligned}$$

we get

$$\begin{aligned} I(M, U_x, X^t \wedge U_y, Y^t | Z^t, F^{t-1}) \\ = I(M, U_x, X^t \wedge U_y, Y^{t-1} | Z^t, F^{t-1}) \\ \leq I(M, U_x, X^t \wedge U_y, Y^{t-1} | Z^{t-1}, F^{t-1}) \\ = I(M, U_x, X^{t-1} \wedge U_y, Y^{t-1} | Z^{t-1}, F^{t-1}) \\ = 0, \end{aligned}$$

where the first equality and inequality follow since  $Y_t$  and  $Z_t$ , respectively, are outputs of  $V_1$  for input  $Z_t$  and  $V_2$  for input  $X_t$ , and the second equality holds since  $X_t = e_t(M, U_x, F^{t-1})$ , which completes the proof.  $\square$

*Proof of Theorem 6.* Given an  $(N, n, \epsilon, \delta)$  wiretap code, a message  $M \sim \text{unif}\{1, \dots, N\}$  and its decoded value  $\hat{M} = d(Y^n)$  satisfy the conditions for Theorem 5 with  $K = M$ ,  $\hat{K} = \hat{M}$ , and  $Z = (Z^n, \mathbf{F})$ . Letting  $Q_{M\hat{M}Z^n\mathbf{F}}$  be the distribution on  $(M, \hat{M}, Z^n, \mathbf{F})$  when the underlying

channel is  $V$ , by Lemma 7 and Theorem 5 we get

$$\log N \leq -\log \beta_{\epsilon+\delta+\eta}(P_{MMZ^n\mathbf{F}}, Q_{MMZ^n\mathbf{F}}) + 2 \log \frac{1}{\eta}.$$

Note that a test for the simple binary hypothesis testing problem for  $P_{MMZ^n\mathbf{F}}$  and  $Q_{MMZ^n\mathbf{F}}$  along with the wiretap code constitutes an active hypothesis test for  $W$  and  $V$ . Therefore,

$$\begin{aligned} & -\log \beta_{\epsilon+\delta+\eta}(P_{MMZ^n\mathbf{F}}, Q_{MMZ^n\mathbf{F}}) \\ & \leq -\log \beta_{\epsilon+\delta+\eta}(W, V, n), \end{aligned}$$

which completes the proof.  $\square$

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